

MA 442 - Quiz

April 24

Name: _____ BUID: _____

Take home quiz. Due April 24

Question 1. Let

$$A \stackrel{\text{def}}{=} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

(a) Define the constants a_n, b_n by the formula

$$\begin{bmatrix} a_n \\ b_n \end{bmatrix} \stackrel{\text{def}}{=} A^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \quad (1)$$

Show that $b_0 = 0$ and $a_n = b_{n+1}$ for all $n \geq 0$.

(b) Continuing as in part (a), show that

$$A^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} b_{n+1} \\ b_n \end{bmatrix} \quad (2)$$

and that

$$b_{n+2} = b_{n+1} + b_n. \quad (3)$$

(c) Diagonalize A .

(d) Use (c) to obtain a formula for b_n .

(e) Compute

$$\lim_{n \rightarrow \infty} \frac{b_{n+1}}{b_n}. \quad (4)$$