MA 442 - Fake Quiz

January 21

Name: ________________________________ **BUID:** ____________________

Solve **both** of the following two questions.

Question 1. Consider the vector space

$$\mathrm{V} = \mathcal{F}(\{0, 1, 2\}, \mathbb{R})$$

of all functions from the three element set $\{0, 1, 2\}$ to the real numbers. (We defined the vector space structure in discussion.) Consider the functions $f, g, h \in V$ defined by $f(t) = t+1, g(t) = t^3 - 3t^2 + 3t + 1, h(t) = 2t + 2$.

(a) Show that $f = g$ in V.

(b) Show that $f + g = h$ in V.

(a) Since $f(0) = g(0) = 1$, $f(1) = g(1) = 2$ and $f(2) = g(2) = 3$ it follows that f and g define the same functions $\{0, 1, 2\} \longrightarrow \mathbb{R}$.

(b) Again, by direct calculation we see that $(f + g)(0) = h(0) = 2, (f + g)(1) = h(1) = 4, (f + g)(2) = h(2) = 6$.

Question 2. Let V be the set of all functions $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(1) = 0$. Show how V can be given the structure of a vector space. (You must define addition and scalar multiplication and then justify the axioms of a vector space.)

We use the rules of addition and scalar multiplication inherited from viewing V as a subset of $\mathcal{F}(\mathbb{R}, \mathbb{R})$. Note that this latter set has vector space structure that we reviewed in discussion. We need to check that these rules are well-defined on V. Indeed, suppose that $f, g \in \mathrm{V}$. We need to see that $f + g$ is also an element of V: indeed, $(f + g)(1) = f(1) + g(1)$ by definition. But, since $f(1) = g(1) = 0$ it follows that $(f + g)(1) = 0$. Thus $f + g \in \mathrm{V}$. Similarly, we see that if $\lambda \in \mathbb{R}$ and $f \in \mathrm{V}$ then $(\lambda f)(1) = \lambda f(1) = \lambda \cdot 0 = 0$. Thus $\lambda f \in \mathrm{V}$. Finally, to see that V is a vector space we need to make sure that the zero vector is in V. The zero vector in $\mathcal{F}(\mathbb{R}, \mathbb{R})$ is the zero function $\mathbf{0}$. This is the function $\mathbf{0}(t) = 0$ for all $t \in \mathbb{R}$. The zero function certainly satisfies $\mathbf{0}(1) = 0$, so $\mathbf{0} \in \mathrm{V}$.

In fact, we have shown that V is a *subspace* of $\mathcal{F}(\mathbb{R}, \mathbb{R})$.