MA 442 - Quiz

March 18

Name: ______________________ **BUID:** ______________________

There are two questions, you must answer both of them to receive full credit.

Question 1. Suppose that A is a diagonal 2×2 matrix. That is, one of the form

$$A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \tag{1}$$

for some scalars a_1, a_2. Prove or find a counterexample to the following statement

"*For every* 2×2 *matrix* B *one has* $AB = BA$ ".

The statement is false. Consider the matrices

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}. \tag{2}$$

Then

$$AB = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = BA. \tag{3}$$

Question 2. A *left inverse* of a linear transformation $\mathsf{T}\colon \mathsf{V} \longrightarrow \mathsf{W}$ is a linear transformation $\mathsf{L}\colon \mathsf{W} \longrightarrow \mathsf{V}$ such that $\mathsf{L} \circ \mathsf{T} = \mathbb{1}_{\mathsf{V}}$. Show that if T has a left inverse L then $\dim \mathsf{V} \leq \dim \mathsf{W}$.

Suppose that $\mathsf{T}(x) = \mathsf{T}(y)$. Apply L to both sides to get

$$x = \mathsf{L}(\mathsf{T}(x)) = \mathsf{L}(\mathsf{T}(y)) = y. \tag{4}$$

Thus, T is injective. So, by the dimension theorem we conclude that $\dim \mathsf{V} \leq \dim \mathsf{W}$.