Syllabus for MA 563
Introduction to Differential Geometry

Course title from registrar

MA 563, section A1. Introduction to Differential Geometry.

Instructor

Brian R. Williams. Office: CDS 411. Email: bwill22@bu.edu.

- Office hours: TBA.
- Website: https://brianrwilliams.github.io/

Time and location

- Lecture times: TuTh 2:00pm–3:15pm
- Lecture location: CAS 214

Course summary

This is a proof-based introduction to differential geometry. We begin with the classical geometry of curves and surfaces in Euclidean space, developing the geometric meaning of curvature through concrete examples. We then study the intrinsic geometry of surfaces, including covariant differentiation, geodesics, parallel transport, and the Gauss–Bonnet theorem. In the final part of the course, we introduce smooth and Riemannian manifolds and explain how the geometry of surfaces extends to higher dimensions. Topics include, but are not limited to:

- regular curves, arclength, curvature, torsion, and the Frenet–Serret equations;
- parametrized surfaces, tangent planes, and the first fundamental form;
- the Gauss map, shape operator, principal curvatures, and the second fundamental form;
- Gaussian and mean curvature, ruled surfaces, and minimal surfaces;
- covariant differentiation, parallel transport, geodesics, and geodesic curvature;
- the Theorema Egregium, the Gauss–Codazzi equations, and the Gauss–Bonnet theorem;
- smooth manifolds, tangent spaces, Riemannian metrics, connections, and the curvature tensor.

Textbook

Wolfgang Kühnel, *Differential Geometry: Curves–Surfaces–Manifolds*, Third Edition, Student Mathematical Library, Volume 77, American Mathematical Society, 2015.

Prerequisites

Official course prerequisites: MA 225 (or MA 230), MA 226 (or MA 231), and MA 242 (or MA 442). Students should be comfortable with multivariable calculus and linear algebra. The course relies on rigorous mathematical proof, so prior experience reading and writing proofs is strongly recommended.

Problem sets

An ungraded problem set will be distributed approximately every two weeks. Each set will ordinarily contain about five to seven problems. These problems are an essential source of practice: the examinations will be at a comparable level. Students are strongly encouraged to discuss the problems with one another, but should make sure that they can write and explain complete solutions independently. Problem-set solutions and/or discussion of selected problems will be provided.

If you are struggling to solve the homework problems then please make sure to ask questions and/or attend office hours. The examinations will cover the same material as the homework sets and will be at a comparable level.

Assessment

- **Midterm exams.** There will be two in-class midterm exams.
  - Midterm exam 1: **Thursday, October 8**.
  - Midterm exam 2: **Thursday, November 19**.

  Each midterm exam must be completed individually and without any aid from devices, notes, or textbooks. Students can use one "formula sheet". There are **no make-up midterm exams** under any circumstances outside of medical emergencies.
- **Group project.** Students will work in groups of two on an extension of material from the course. Each group will submit a mathematical expository paper and give a short presentation. The project expectations, suggested topics, and rubric appear in the separate project handout. Project deadlines and presentation dates are TBA.
- **Final exam.** There will be a cumulative two-hour final exam occuring during the university-wide final-examination period whose date and time follow the *Final Exam Matrix* posted by the registrar. According to this matrix, the date and time for our final exam is:

  Tuesday, December 15 | 3:00pm–5:00pm

Grade breakdown

First midterm	20%
Second midterm	20%
Group project	25%
Final exam	35%
Total	100%

The biweekly problem sets are ungraded and do not contribute directly to the numerical course grade.

How to succeed in this class

- Attend class and work steadily. Differential geometry combines geometric intuition, calculation, and proof; each of these improves through regular practice.
- Treat the ungraded problem sets as required preparation. Attempt problems before consulting solutions, and practice writing complete arguments without notes.
- If you are struggling, reach out by email or attend office hours early in the semester. October 7 is the last day to drop this class without a "W" grade. November 6 is the last day to drop this class with a "W" grade.
- Additional academic-support resources are available through the Department of Mathematics and Statistics and the Educational Resource Center; current details are available at `https://www.bu.edu/math/` and `https://www.bu.edu/erc/`.

This syllabus is subject to minor changes. Any change made after the first day of class will be announced and reflected in the updated syllabus.