HOMEWORK 5
DUE OCTOBER 13

There are two problems to turn in.

(1) This problem utilizes notation and constructions from problem 2 of homework 3. Define $h_w \colon U_w \to \mathbf{R}$ by the formula
$$h_w([z:w]) = \frac{|z/w|^2 - 1}{|z/w|^2 + 1}.$$
(You should think for a moment about why h_w is well-defined). Also, define $h_z \colon U_z \to \mathbf{R}$ by the formula
$$h_z([z:w]) = -\frac{|w/z|^2 - 1}{|w/z|^2 + 1}.$$

(a) Show that there is a unique smooth function $h \colon \mathbf{CP}^1 \to \mathbf{R}$ such that $h|_{U_w} = h_w$ and $h|_{U_z} = h_z$.
(b) Classify the critical values of h.
(c) If $c \in \mathbf{R}$ is a regular value of h, show the the level set $h^{-1}(c)$ is diffeomorphic to S^1.
(d) For $c \in \mathbf{R}$ a critical value of h, describe the set $h^{-1}(c)$.

(2) Let $f \colon \mathbf{R}^n \setminus \{0\} \to \mathbf{R}$ be a smooth function such that for all $\lambda > 0$ one has
$$f(\lambda x) = \lambda^c f(x)$$
for some real number c. Let
$$E = \sum_i x^i \frac{\partial}{\partial x_i}$$
be the Euler vector field. Show that
$$Ef = cf.$$