MATHEMATICS 721
INTRODUCTION TO DIFFERENTIAL TOPOLOGY I
FALL 2023

Course name.

MA 721 A1—Introduction to Differential Topology I. Fall 2023.

Instructor.

Brian R. Williams. Office: CDS 411. Email: bwill22@bu.edu.

Time and location.

Monday, Wednesday, Friday 12:20 PM — 1:10 PM in WED Room 212. (This is the Wheelock College of Education and Human Development located on 2 Silber Way.

Office hours.

TBA.

Course content and summary.

A smooth manifold is a topological space which locally 'looks like' euclidean space. Besides their importance within the realm of pure mathematics, smooth manifolds appear naturally in many applications. They are especially prevalent in theoretical physics including, but not limited to, the contexts of classical mechanics, general relativity, and gauge theory.

Differential topology is the study of the properties and structures of smooth manifolds. In this course, we will introduce smooth manifold, maps between smooth manifolds, tangent bundles and vector fields, cotangent bundles, vector bundles, immersions and submersions, tensors, Lie groups and Lie algebras, orientations, differential forms and integration, de Rham cohomology, integral flows, Lie derivatives, and foliations.

This is a fast-paced class and it is essential that students keep a regular attendance. In the event that the student must miss class they should arrange to get the notes they missed.

Textbook.

Introduction to Smooth Manifolds, 2nd edition, by John M. Lee; Springer; ISBN-13:1441999818. An up-to-date erratum can be found here https://sites.math.washington.edu/~lee/Books/ISM/errata.pdf.

Course website. The course website is

https://brianrwilliams.github.io/ma721/index.html.

Grades will be posted on Blackboard.

Prerequisites.

Multivariable calculus, linear algebra, and basic concepts from topology.

Assessment.

You will be assessed through homework and a final exam. You will have one week to complete each homework assignment; it will be assigned on Friday and will be due the following Friday at class time. **No late homework will be accepted.** Grades are determined by the formula

$$\text{Final grade} = \frac{2}{3}(\text{homework average}) + \frac{1}{3}(\text{final exam}).$$

Approximate weekly schedule

The following weekly schedule is subject to change.

- Week 1, Sep. 6–8. Topological manifolds, smooth structures.
- Week 2, Sep. 11–15. Smooth maps, partitions of unity.
- Week 3, Sep. 18–22. Tangent space, derivatives, tangent bundle.
- Week 4, Sep. 25–29. Embeddings, immersions, and submersions.
- Week 5, Oct. 2–6. Submanifolds, Sard's theorem.
- Week 6, Oct. 10–13. Lie groups, Lie algebras, vector fields.
- Week 7, Oct. 16–20. Vector bundles, cotangent bundle.
- Week 8, Oct. 23–27. Tensors, differential forms.
- Week 9, Oct. 30–Nov. 3. Integral flows, Lie derivatives.
- Week 10, Nov. 6–10. Distributions, foliations.
- Week 11, Nov. 13–17. Orientations, integration.
- Week 12, Nov 20. Stoke's theorem.
- Week 13, Nov. 27–Dec. 1. De Rham cohomology.
- Week 14, Dec. 4–8. Distributions and foliations (cont.).
- Week 15, Dec. 12. The de Rham theorem.