MA 726 — DIFFERENTIAL GEOMETRY II

Course name. MA 726–Differential Geometry, II.

Time and location. Tuesday and Thursday from 12:30 PM – 1:45 PM in MCS B31.

Summary.

The topic for this semester is the index theory of elliptic operators. The index of an elliptic differential operator is defined analytically, in the sense that it counts numbers of solutions to a differential equation. Gelfand observed that the index is actually homotopy invariant and found expressions for the index, in many examples, in terms of topological quantities like characteristic classes. Examples of such 'index theorems' include the Hirzebruch–Riemann–Roch theorem and the Hirzebruch signature formula. Atiyah and Singer's foundational work gave a general proof of the index theorem for elliptic operators using K-theory. Decades later, a completely geometric proof of the Atiyah–Singer index theorem using the heat equation was presented by Atiyah, Bott, Patodi, and Gilkey; and later refined by Getzler, Witten, and Alvarez–Gaume.

In this course we will present the heat kernel approach to the index theorem. We will begin with a review of some of the background in basic differential geometry including connections, characteristic classes, and the Thom isomorphism theorem. From here, the main topics include superconnections, ellipticity, heat kernels, spinors, Dirac operators, zeta-functions, determinant line bundles, and further topics, time permitting.

Textbooks and recommended readings.

We will primarily follow the textbook:

- *Heat kernels and Dirac operators*, by Berline, Getzler, and Vergne.

Other textbook (or textbook-like) references include:

- *The Laplacian on a Riemannian manifold*, by Rosenberg.
- *The Aityah–Singer index theorem*, by Freed.
- *Notes on the Atiyah–Singer index theorem*, by Nicolaescu.

Prerequisites.

This is the second semester in differential geometry. While the first semester (MA 725) is not formally required, the student is expected to have had thorough exposure to the theory of smooth manifolds along the lines of the sequence MA721-722. This is an extremely fast paced class and students are expected to attend every class. If you are falling behind with the material please contact me immediately.

Assessment.

There will be **four** homework assignments. Your final grade will be completely determined by your highest **three** homework assignment scores (all weighted equally):

$$\text{Final grade} = \frac{1}{3}\left(x_1 + x_2 + x_3 + x_4 - \min\{x_1, x_2, x_3, x_4\}\right), \tag{1}$$

where x_i is your percentage score on the ith homework.

No late homework will be accepted or graded.