Last time we showed that

$$\lim_{x\to\infty} \frac{\ln x}{x} = 0. \tag{136}$$

This kind of limit suggests insight into the 'growth rate' of functions. We say that a function g **grows faster** than a function f if

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0. \tag{137}$$

Equivalently, $\lim_{x\to\infty} g(x)/f(x) = \infty$. We say that the growth rates of f, g are **comparable** if the limit $\lim_{x\to\infty} f(x)/g(x)$ is a finite nonzero number.

Example 2.55. In the last example, we saw that x grows faster than $\ln x$.

Example 2.56.

- Are there any $p, q > 0$ such that $(\ln x)^p$ and x^q have comparable growth?
- Are there any $p, q > 0$ such that e^{px} and x^q have comparable growth?