OCTOBER 14, 2022

Today we will study the derivative of the arctangent function $\arctan x$. Like arcsine, arctangent is defined as the inverse of a function. Can you guess which one?

We consider the tangent function $\tan x$ on the interval

$$-\frac{\pi}{2} < x < \frac{\pi}{2}. \tag{82}$$

On this interval, the function $\tan x$ is one-to-one.

Example 2.32. What is the range of $\tan x$ on the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$?

On this interval the inverse $\arctan x$ exists and satisfies

$$\tan(\arctan x) = x \tag{83}$$

for all x.

Example 2.33. Use the substitution $y = \arctan x$ and implicit differentiation to find the derivative of arctan.

We just found that.

Proposition 2.34. *The derivative of the function* arctan x *is*

$$(\arctan x)' = \frac{1}{1+x^2}. \tag{84}$$

Example 2.35. Find the equation of the line tangent to the graph of the function

$$f(x) = \arctan(\ln x) \tag{85}$$

at the point $x = e^2$.

Example 2.36. Suppose an airplane takes off from the airport and is moving horizontally 5 kilometers above the ground at constant speed of 16 kilometers per hour. What is the rate of change of the angle of elevation of the airplane relative to the airport when the airplane is directly above a point which is 10 kilometers away from the airport? Give your answer in radians per hour.