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We've learned what the derivative tells us about the shape of a graph. What about the second derivative? This controls the feature called *concavity.*

- A function is **concave up** on an interval if $f'(x)$ is increasing on the interval.
- A function is **concave down** on an interval if $f'(x)$ is decreasing on the interval.
- A point $x = c$ where f changes concavity is called an **inflection point.**

An easy reformulation of the first two points is the following:

- If $f''(x) > 0$ on some interval, we say that f is concave up on that interval.
- If $f''(x) < 0$ on some interval, we say that f is concave down on that interval.
- If $f''(c) = 0$ and $f''(c)$ changes sign as we go through $x = c$ then $x = c$ is an inflection point of the function.

Example 2.41. Describe the concavity and the inflection points of the following functions.

(a) $f(x) = \arctan(x)$.
(b) $f(x) = \sqrt{x} \ln x$ on the domain $x > 0$.

Next we move on to the second derivative test.

Suppose that $x = c$ is a critical point of a function f which satisfies $f'(c) = 0$.

- If $f''(c) > 0$ then f has a local minimum at $x = c$.
- If $f''(c) < 0$ then f has a local maximum at $x = c$.

If $f''(c) = 0$ then the test is inconclusive—the function may have a local min/max or neither.

Example 2.42. Use the second derivative test to locate and characterize the local extrema of the function

$$f(x) = \frac{e^x}{x+1} \tag{91}$$

defined for $x \neq 1$.