

1. SEPTEMBER 23,2022

Today we'll discuss more examples of derivatives.

Examples.

Example 1.1. Let

(13)
$$f(x) = \sqrt{x+7}.$$

- (a) Is the function $f(x)$ differentiable? If so, on what domain?
- (b) Use the limit definition to compute the derivative of $f(x)$ at $x = 2$.
- (c) Find the equation of the line tangent to the graph of the function $f(x)$ at $x = 2$.

Here is an example of a function which fails to be differentiable at a point.

Example 1.2. Consider the absolute value function $f(x) = |x|$. At which point is $f(x)$ not differentiable. For the values of x that $f(x)$ is differentiable, find $f'(x)$.

Properties of derivatives.

- (Constants) The derivative of a constant function $c(x) = c$ is identically zero, $c'(x) = 0$. If c is any constant and f is a function then

$$(14) \quad (cf)' = cf'.$$

- (Sum) The derivative of a sum is the sum of derivatives:

$$(15) \quad (f + g)' = f' + g'.$$

- (Power rule) For any integer n the derivative of $f(x) = x^n$ is $f'(x) = nx^{n-1}$. In fact, if $r = p/q$ is a rational number then we still have

$$(16) \quad (x^r)' = rx^{r-1}.$$

For example, the derivative of the function \sqrt{x} is $1/\sqrt{x}$.

Warning: The derivative of the *product* $f \cdot g$ is *not* the product of the derivatives:

$$(17) \quad (f \cdot g)' \neq f' \cdot g'.$$

Soon, we will learn rules for computing the derivative of the product of two functions.

Example 1.3. Where is the function $f(x) = x^{2/3} + 7x^4$ differentiable? For the values of x that f is differentiable, what is $f'(x)$?