HOMEWORK 1
DUE SEPTEMBER 15

(1) Endow the cross

$$+$$

with the subspace topology in $\mathbf{R}^2$ (meaning that a set $U \subset +$ is open if and only if it is the intersection of an open set $\widetilde{U} \subset \mathbf{R}^2$ with $+ \in \mathbf{R}^2$). Show that the cross is not locally Euclidean.

(2) Endow

$$\bigvee$$

with the subspace topology in $\mathbf{R}^2$. (For concreteness, you can think of this space as the graph of the function $x \mapsto |x|$.) Is $\bigvee$ a topological manifold? Is it a smooth manifold? (You must provide justification to receive full credit.)

(3) Suppose that $\{(U_\alpha, \phi_\alpha)\}_{\alpha \in I}$ and $\{(V_\beta, \psi_\beta)\}_{\beta \in J}$ are smooth atlases for manifolds M, N of dimensions m, n respectively. Show that

$$\{(U_\alpha \times V_\beta, \phi_\alpha \times \psi_\beta)\}_{\alpha,\beta \in I \times J}$$

is a smooth atlas for the product space $M \times N$. Hence, $M \times N$ is a smooth manifold of dimension $m + n$.