HOMEWORK 8
DUE NOVEMBER 10

There are two problems to turn in.

(1) Let $\pi\colon E \to M$ be a smooth vector bundle of rank k. Suppose $\{U_\alpha\}$ is an open cover of M which is equipped with local trivializations for E:

$$\psi_\alpha\colon E|_{U_\alpha} \to U_\alpha \times \mathbf{R}^k.$$

Let $g_{\alpha\beta}\colon U_\alpha \cap U_\beta \to GL(k, \mathbf{R})$ be the corresponding transition functions. Show that for any $p \in U_\alpha \cap U_\beta \cap U_\gamma$:

$$g_{\alpha\beta}(p) g_{\beta\gamma}(p) = g_{\alpha\gamma}(p).$$

This is called the *cocycle property* for the transition functions $\{g_{\alpha\beta}\}$.

(2) Recall that $\mathbf{CP}^1$ can be identified with the set of lines ℓ in $\mathbf{C}^2$. Define

$$V = \{(\ell, z_1, z_2) \mid \ell \in \mathbf{CP}^1, (z_1, z_2) \in \ell\} \subset \mathbf{CP}^1 \times \mathbf{C}^2.$$

(a) Show that V has the structure of a (real) rank two vector subbundle of the trivial rank four bundle $\mathbf{CP}^1 \times \mathbf{C}^2 = \mathbf{CP}^1 \times \mathbf{R}^4$ over $\mathbf{CP}^1$. (b) Let $\underline{0} \subset V$ be the image of the zero section of the bundle V. Construct a diffeomorphism

$$V \setminus \underline{0} \xrightarrow{\cong} \mathbf{R}^4 \setminus \{0\}.$$