HOMEWORK 9
DUE NOVEMBER 21

Define an equivalence relation on $\mathbf{R}^2$ by the rule

$$(x, y) \sim (x+1, -y) \tag{1}$$

Let $E = \mathbf{R}^2/\sim$ be the quotient space and $q\colon \mathbf{R}^2 \to E$ be the quotient map.

(1) Let $\exp\colon \mathbf{R} \to S^1$ be the map $x \mapsto e^{2\pi i x}$. Show that there exists a unique map $\pi\colon E \to S^1$ such that $\exp \circ p_1 = \pi \circ q$ where $p_1\colon \mathbf{R}^2 \to \mathbf{R}$ is projection onto the first factor.
(2) Endow E with a smooth structure such that q is a smooth submersion.
(3) Endow (E, π) with the structure of a line bundle on S^1.
(4) Show that E is not the trivial line bundle (hint: compute the transition functions).