PRACTICE TRUE/FALSE

(1) An open subset of a smooth manifold is a smooth manifold.

(2) A closed subset of a smooth manifold is a smooth manifold.

(3) If a smooth map $F\colon \mathbf{R}^n \to \mathbf{R}^m$ satisfies $(\mathrm{d}F)_a = F$ for every $a \in \mathbf{R}^n$ then F is a linear map.

(4) If the graph of a function $f\colon M \to \mathbf{R}$ is a smooth submanifold of $M \times \mathbf{R}$, then f is smooth.

(5) For any vector field X one has $[X, X] = 0$.

(6) If X and Y are vector fields on $\mathbf{R}^n$ such that $[X, Y] = 0$ then $X = \lambda Y$ for some constant λ.

(7) Let $F\colon M \to N$ be smooth. Then $F^{-1}(c)$ is a smooth submanifold of M if and only if c is a regular value of F.

(8) An injective smooth immersion is a smooth embedding.

(9) If F is an injective smooth map of constant rank then it is an embedding.

(10) Let $F\colon M \to N, G\colon N \to P$ be submersions. Then $G \circ F\colon M \to P$ is a submersion.

(11) Let $F\colon M \to N, G\colon N \to P$ be immersions. Then $G \circ F\colon M \to P$ is an immersion.

(12) A local diffeomorphism is an injective map.

(13) The set

$$\{(x, y) \in \mathbf{R}^2 \mid |x|^3 + |y|^3 = 1\}$$

is a smooth submanifold of $\mathbf{R}^2$.

(14) If V is a vector space, then there is a canonical linear isomorphism $T_v V \cong V$ for any $v \in V$.

(15) If G is a Lie group then the sets of left-invariant vector fields on G and right-invariant vector fields on G are in bijective correspondence.

(16) A manifold M of dimension n is parallelizable if and only if there exists a diffeomorphism $TM = M \times \mathbf{R}^n$.