November 6

Gluing data: • $\{U_\alpha\}$ cover for M.

• $E = \coprod_{p \in M} E_p$, where E_p is dim $= k$ v.s..

• $\psi_\alpha : \pi^{-1}(U_\alpha) \xrightarrow{\cong} U_\alpha \times \mathbb{R}^k$ are bijections

• $g_{\alpha\beta} : U_\alpha \cap U_\beta \to GL(k, \mathbb{R})$ s.t.

$$\psi_\alpha \circ \psi_\beta^{-1}(p, v) = (p, g_{\alpha\beta}(p)\, v).$$

Prop: E has the unique str. of a v.b. on M.

Pf: (Sketch) Find charts for E. For $p \in M$ let U_α contain p. Let (V_p, ϕ_p) be a chart for M containing p, s.t.

$$V_p \subset U_\alpha.$$

Define chart for E:

$$\pi^{-1}(V_p) \xrightarrow{\Psi_\alpha} V_p \times \mathbb{R}^k \xrightarrow{\phi_p \times \mathbb{1}} \mathbb{R}^n \times \mathbb{R}^k .$$

Also need to check Hausdorff... $\boxed{//}$

—— // ——

A smooth **section** of v.s. $\begin{matrix} E \\ \downarrow \pi \\ M \end{matrix}$ is a smooth

$$s : M \longrightarrow E$$

s.t. $\pi \circ s = \mathbb{1}_M$. $(*)$

A **local section** defined on open $U \subset M$ is

$$s : U \longrightarrow E$$

s.t. $\pi \circ s = \mathbb{1}_U$. $(**)$

We call a map satisfying (α) or ($\alpha\alpha$) a "rough" section (so, not necessarily smooth.)

The ___support___ of section s is the closed set

$$\{ p \in M \mid s(p) \neq 0 \} \subset M.$$

The ___zero section___ is

$$0 : M \longrightarrow E, \quad p \longmapsto 0 \in E_p.$$

___Ex___: A section of the trivial v.b.

$$\begin{tikzcd} M \times \mathbb{R}^k \arrow[d] \\ M \arrow[u, bend left, "s"] \end{tikzcd}$$

is simply a smooth map

$$s : M \longrightarrow \mathbb{R}^k.$$