Last time we introduced the left Riemann sum approximation to area under a curve. Let's now look at variants of this: the right and midpoint Riemann sums.

The situation is that we have a function $y = f(x)$ defined on some interval $[a, b]$. The steps to approximate the area are as follows.

(1) Determine the number subintervals n that we will partition the interval $[a, b]$ into. The length of a subinterval is

$$\Delta x = \frac{b-a}{n}. \tag{150}$$

(2) Draw n rectangles as follows:
   - In the **left** Riemann sum, the top left corner of each rectangle touches the function. So you would draw n rectangles of width Δx and of heights

$$f(a), f(a+\Delta x), f(a+2\Delta x), \ldots, f(a+(n-1)\Delta x) = f(b-\Delta x). \tag{151}$$

   - In the **right** Riemann sum, the top right corner of each rectangle touches the function. So you would draw n rectangles of width Δx and of heights

$$f(a+\Delta x), f(a+2\Delta x), f(a+3\Delta x), \ldots, f(a+n\Delta x) = f(b). \tag{152}$$

   - In the **midpoint** Riemann sum, the center of each rectangle touches the function. So you would draw n rectangles of width Δx and of heights

$$f(a+\frac{1}{2}\Delta x), f(a+\frac{3}{2}\Delta x), f(a+\frac{5}{2}\Delta x), \ldots, f(b-\frac{1}{2}\Delta x). \tag{153}$$

(3) The last step is to sum the areas of the rectangles.

Example 2.66.

46. Displacement from a table of velocities The velocities (in m/s) of an automobile moving along a straight freeway over a four-second period are given in the following table.

t (s)	0	0.5	1	1.5	2	2.5	3	3.5	4
v (m/s)	20	25	30	35	30	30	35	40	40

Use the midpoint Riemann sum approximation with four subintervals to approximate the distance traveled by the automobile from $t = 0$ to $t = 4$.