Let's begin with an example related to logarithms.

Example 2.31. Find the slope of the line tangent to the graph of $f(x) = x^x$ at $x = 1$. Does this graph have any horizontal tangent lines?

We move onto derivatives of inverse trigonometric functions. Recall that two functions $f(x), g(x)$ are said to be inverses of one another if

$$f(g(x)) = x, \quad \text{and} \quad g(f(x)) = x. \tag{82}$$

Thus, the defining properties of the the inverse sine function arcsin is, for instance, is

$$\sin(\arcsin x) = x, \quad \text{and} \quad \arcsin(\sin x) = x. \tag{83}$$

An important caveat here is that the inverse of a function is strictly only define when the function is one-to-one. This means that the original function must pass the *horizontal* line test. Clearly, the sine function fails this as for any fixed y_0 the equation $y_0 = \sin x$ has infinitely many solutions. One way around this is to restrict the domain of $\sin x$ so that it does pass the horizontal line test. A standard choice is the interval $[-\pi/2, \pi/2]$. The range of $\sin x$ in this domain is $[-1, 1]$.

Therefore, the domain of $\arcsin x$ is $[-1, 1]$, and its range is $[-\pi/2, \pi/2]$.

Example 2.32. What is $\arcsin(\pm 1)$? What is $\arcsin(1/\sqrt{2})$?

Just the knowledge of the properties in (83) will allow us to nail down the derivative of $\arcsin x$ using the rules that we know. First, introduce a dependent variable $y = \ln x$ into the first expression above:

$$\sin y = x. \tag{84}$$

Next, we take the implicit derivative with respect to x to get

$$y' \cos y = 1. \tag{85}$$

In other words,

$$y' = \frac{1}{\cos y}. \tag{86}$$

This last expression is not so satisfying since the right hand side involves the variable y. We would like to go back to the original equation $\sin y = x$ to solve for y, but this is also not in an immediately useful form.

The problem boils down to the following: if y is an angle such that $\sin y = x$. What is the value of $\cos y$? Draw a right triangle with one angle given by y and whose hypotenuos is unit length. Then, since $\sin y = x = x/1$, we know that the length of the side opposite to the angle y is x. The value of $\cos y$ is simply the length of the last side, which by the Pythagorian theorem we know to satisfy

$$\cos^2 y + x^2 = 1 \tag{87}$$

In other words.

$$\cos y = \frac{1}{\sqrt{1-x^2}}. \tag{88}$$

Remember, this is only defined on the interval $(-1, 1)$. We arrive at.

Proposition 2.33. *The derivative of* $\arcsin x$ *on the interval* $(-1, 1)$ *is*

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}. \tag{89}$$

Example 2.34. Consider the function $f(x) = \arcsin\left(\frac{x^2}{4}\right)$. Describe all horizontal and vertical tangent lines.