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We now turn to word problems involving at least two parameters which change with respect to time. For example:

Two airplanes approach an airport with known speeds, one flying west and one flying north. How fast is the distance between them changing?

Here are some steps to take to organize and solve such a problem:

1. Sketch the problem (usually there is some geometry going on).
2. Write down a few equations that express relationships between the parameters at hand.
3. Treat these equations as implicit equations, and differentiate to find relationships among the rates (with respect to time).
4. Substitute known quantities.
5. Check (and double check) units.

Let's put some teeth into the above example.

Example 2.37. Suppose the westerly moving airplane is moving at 120 mi/hr and the northerly moving airplane is moving at 150 mi/hr. Assuming they fly at a constant speed and elevation, how fast is the distance between them changing when the westerly moving airplane is 180 miles from the airport and the northerly moving one is 225 miles from the airport?

Example 2.38. A boat is being pulled towards the dock by a rope that attaches to the back of the boat and a crank on the dock. The height of the crank is 10 meters above sea level and the rope is being cranked at a rate of 4 meters per minute.

- How fast is the boat approaching the dock when it is at a distance of 6 meters away from the bottom of the dock (in the horizontal direction).
- How fast is the angle between the rope and the water changing at the same moment?

Example 2.39. Kate is flying a kite. Once Kate's kite reaches a height of 50 feet above her hands, it rises no higher but drifts horizontally in a wind blowing 5 feet per second. How fast is the string running through Kate's hands at the moment when she has released 120 feet of string?