OCTOBER 5, 2022

Today we will continue with more examples involving the chain rule. Recall that the chain rule says that if $h = f \circ g$ is a composition of two functions then

$$h'(x) = f'(g(x)) \cdot g'(x). \tag{56}$$

Example 2.16. A function $f(x)$ is *even* if $f(-x) = f(x)$ and is *odd* if $f(-x) = -f(x)$.

- If f is even, is the derivative f' even, odd, or neither?
- If f is odd, is the derivative f' even, odd, or neither?

Example 2.17. Express the function

$$h(x) = \frac{1}{(2x^2+3)^{10}} \tag{57}$$

as a composition of two functions. Compute $h'(x)$ using chain rule.

Let's see an example of chain rule where we do not necessarily know the exact form of one of the functions involved.

Example 2.18. Suppose that f is differentiable and satisfies $f'(0) = 2$, $f'(1/2) = 1/3$. Let

$$h(x) = f(\sin x). \tag{58}$$

Find $h'(0)$ and $h'(\pi/6)$.

Example 2.19. Let

$$h(x) = x\sqrt{5 - x^2}. \tag{59}$$

(a) Find $h'(x)$.
(b) Determine the equation of the lines tangent to the graph at the values $x = 1$ and $x = -2$.
(c) At which points (if any) do the lines in part (b) intersect? Express your answer as an ordered pair.

Also, chain rule can be used to find limits.

Example 2.20. Find the following limit (if it exists)

$$\lim_{h\to 0} \frac{\sqrt{4+\sin(h)}-2}{h}. \tag{60}$$