Today we will review material in sections 2.4–2.5 of the book.

Recall that we say

$$\lim_{x \to a} f(x) = \infty \tag{1}$$

if we can make the value $f(x)$ be infinitely large by choosing x to be sufficiently close to a. Similarly, we can define

$$\lim_{x \to a} f(x) = \pm\infty, \quad \lim_{x \to a^{\pm}} f(x) = \pm\infty. \tag{2}$$

In any of these cases we call the line $x = a$ a vertical asymptote of the function f.

0.0.1. Challenge question. If $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = -\infty$ then what can we say about $\lim_{x \to a}(f(x) + g(x))$?

Example 0.1. Compute the following limit (if it exists)

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x^2} \right) \tag{3}$$

Example 0.2. Compute the following limit (if it exists)

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{2x+1}{x} \right). \tag{4}$$

Example 0.3. Define the function

$$f(x) = \frac{x^2 - x - 2}{x - a} \tag{5}$$

where a is some number. For which values of a do the following limits exist.

- $\lim_{x \to a^+} f(x)$.
- $\lim_{x \to a^-} f(x)$.
- $\lim_{x \to a} f(x)$.

0.0.2. Recall that we say

$$\lim_{x\to\infty} f(x) = L \tag{6}$$

if we can make $f(x)$ as close to the value L for x a large enough number. In this case we say that $y = L$ is a horizontal asymptote of the function f.

Example 0.4. Find the limit (if it exists)

$$\lim_{x\to\infty} \frac{x^2 + 2x + 1}{4x^2 + 5} \tag{7}$$

Example 0.5. Find the limit (if it exists).

$$\lim_{x\to\infty} \frac{e^x \sin x}{e^{2x} + 1} \tag{8}$$