Definition 0.6. Suppose that a function f is defined on an open subset of $\mathbf{R}$ which contains the point a. We say that f is continuous at a if

$$f(a) = \lim_{x \to a} f(x). \tag{9}$$

0.0.3. Examples and non-examples of continuous functions .

0.0.4. Checklist for continuity. For a function f to be continuous at a point a, the following three conditions must hold.

(1) f must be defined at a.
(2) the limit $L = \lim_{x \to a} f(x)$ must exist.
(3) the limit L must equal $f(a)$.

Example 0.7. At which points $a \in \mathbf{R}$ is the function $f(x) = x/|x|$ continuous?

0.0.5. Properties of continuous functions. We say a function is continuous if it is continuous at all points in its domain.

(1) the sum, product, difference, and ratio (when defined) of two continuous functions is again continuous.
(2) all polynomials are continuous.
(3) Rational functions (ratios of polynomials) are continuous at every point in their domain.
(4) the composition of two continuous functions is again continuous.

0.0.6. Continuity on closed intervals. So far we've covered what it means for a function to be continuous on domains which are open intervals, like $(0, \infty)$. What about the function $f(x) = \sqrt{x}$ which is defined on the closed interval $[0, \infty)$? The problem is at the closed point $0 \in [0, \infty)$. In addition to checking continuity on the open interval $(0, \infty)$ we also need to make sure that the right sided limit at 0 agrees with value of the function at that point

$$\lim_{x \to 0^+} f(x) \stackrel{?}{=} f(0). \tag{10}$$

If this is the case then we say that f is continuous on the closed interval $[0, \infty)$.